[This question paper contains 3 printed pages.]

Your Roll No...2022..

Sr. No. of Question Paper : 2780   A

Unique Paper Code   : 62357604

Name of the Paper   : Differential Equations

Name of the Course   : B.A. (Prog.) Mathematics : DSE

Semester   : VI

Duration : 3 Hours   Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions are compulsory.

3. Attempt any **two** parts from each question.

1.

a. Solve :   6

$$(1 + yx)xdy + (1 - yx)ydx = 0$$

b. Solve :   6

$$(y^3 - x^2y)dx + \left(3xy^2 - \frac{x^3}{3}\right)dy = 0$$

c. Solve :   6

$$x + yp^2 - p(1 + xy) = 0$$

d. Show that e^{-2x}, e^{-3x} are the linearly independent solutions of   6
$$y'' + 5y' + 6y = 0.$$
What is the general solution? Find the solution $y(x)$ with the property $y(0) = 0$, $y'(0) = 1$.

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2.

  a. Solve :

$$x^3y' - x^2y = -y^4\sin(x).$$ 6.5

  b. Solve :

$$4y = x^2 + p^2.$$ 6.5

  c. Solve :

$$xy(y - px) = (x + py)$$ 6.5

  d. Show that e^{-x}, xe^{-x}, e^{2x} are the linearly independent solutions of   6.5
$$y''' - 3y' - 2y = 0.$$
What is the general solution? Find the solution y(x) with the property $y(0) = 0$, $y'(0) = 2$, $y''(0) = 3$.

3.

  a. Solve :   6

$$\left(\frac{d^3y}{dx^3}\right) - 3\left(\frac{d^2y}{dx^2}\right) - 6\left(\frac{dy}{dx}\right) = x^2 + 1.$$

  b. Solve :   6

$$(x^2D^2 - 3xD + 5)y = x^2\sin\log x.$$

  c. Apply the method of variation of parameter to solve :   6

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x\,e^x\log x, \qquad x > 0.$$

  d. Solve :   6

$$(D^2 + a^2)y = \cos ax.$$

4.   6.5

  a. Solve the following system of equations :
$$\frac{dx}{dt} + 2y + x = e^t \text{ and } \frac{dy}{dt} + 2x + y = 3e^t.$$ 6.5

  b. Solve :

$$\frac{dx}{x^2 + 2y^2} = \frac{dy}{-xy} = \frac{dz}{xz}.$$ 6.5

  c. Solve :

$$xz^3dx - zdy + 2y\,dz = 0.$$ 6.5

  d. Solve :

$$\frac{dx}{x(y^2 - z^2)} = \frac{dy}{y(z^2 - x^2)} = \frac{dz}{z(x^2 - y^2)}.$$ 6

5.

  a. Eliminate the arbitrary function f from the equation:
$$z = e^{ax+by}f(ax - by)$$ 6
to find the corresponding partial differential equation.

  b. Find the general solution of the differential equation:
$$(x^2 + 2y^2)p - xyq = xz.$$

c. Find the integral surface of the linear partial differential equation 6
$$(x - y)y^2p + (y - x)x^2q = (x^2 + y^2)z$$
Through the curve $xz = a^3$ and $y = 0$.

d. Find the complete integral of the partial differential equation: 6
$$z = px + qy + p^2 + q^2.$$

6.

a. (i) Classify the following partial differential equation into elliptic, parabolic or hyperbolic: 2.5

$$(x - y)(xr - xs - ys + yt) = (x + y)(p - q),$$
where $r = \frac{\partial^2 z}{\partial x^2}$, $s = \frac{\partial^2 z}{\partial x \partial y}$, $t = \frac{\partial^2 z}{\partial y^2}$, $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.

(ii) Eliminate the arbitrary constants a and b from the equation : 4
$$2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$
to find the corresponding partial differential equation.

b. Find the general solution of the differential equation: 6.5
$$px(z - 2y^2) = (z - qy)(z - y^2 - 2x^3).$$

c. Show that the following systems of partial differential equations are compatible and hence solve them 6.5
$$p = x^4 - 2xy^2 + y^4, \qquad q = 4xy^3 - 2x^2y - \sin y.$$

d. Find the complete integral of the partial differential equation: 6.5
$$zpq = p^2(xq + p^2) + q^2(yp + q^2).$$